

90/100

NAME: \_\_\_\_\_

NOTE 1: OPEN BOOK, OPEN NOTES, CLOSED OLD TESTS AND SOLUTIONS.  
NOTE 2: SHOW ALL WORK IN ORDER TO GET FULL CREDIT.

1. 20 pts: Find the inverse Laplace transform of the following function.  
Perform partial fraction expansion.

① 20  
② 30  
③ 40

$$G(s) = \frac{10}{(s+1)^2(s+3)} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{s+3}$$

$$B = (s+1)^2 G(s) \Big|_{s=-1} = \frac{10}{(s+3)} \Big|_{s=-1} = \frac{10}{2} = 5$$

$$A = \left[ \frac{d}{ds} (s+1)^2 G(s) \right]_{s=-1} = \left[ \frac{d}{ds} \left\{ \frac{10}{(s+3)} \right\} \right]_{s=-1} = \left[ \frac{-10}{(s+3)^2} \right]_{s=-1} = \frac{-10}{4}$$

$$C = (s+3) G(s) \Big|_{s=-3} = \left[ \frac{10}{(s+1)^2} \right]_{s=-3} = \frac{10}{4} = \frac{5}{2}$$

$$G(s) = \frac{-\frac{5}{2}}{(s+1)} + \frac{5}{(s+1)^2} + \frac{\frac{5}{2}}{s+3}$$

$$g(t) = \left[ -\frac{5}{2} e^{-t} + \frac{5}{2} e^{-st} + 5 t e^{-t} \right] u(t) \quad \checkmark$$

3. 45 pts. For the transfer function shown below, determine the value of  $K$  so that the Bode magnitude plot intersects at -8 dB at  $\omega = 0.1$  rad/s. Plot the magnitude and phase Bode approximations. Find the 0-dB crossings and label the gain at each break frequency. Find the gain and phase margins.

$$GH(s) = \frac{Ks(s+2)^2(s+100)(s+800)^2(s+5000)}{(s+10)^2(s+40)(s+500)^2(s+1000)(s+2000)^2}$$

The ratio of the constants is:

$$\frac{128 \cdot 10^{10} K}{4 \cdot 10^{20}} = \frac{32}{10^{10}} K$$

$$20 \log\left(\frac{32 K}{10^{10}}\right) = 12$$

$$\log\left(\frac{32 K}{10^{10}}\right) = \frac{12}{20} = 0.6 \Rightarrow \log(3.2 \cdot 10^{-3} K) = 0.6$$

$$3.2 \cdot 10^{-3} K = 10^{0.6}$$

$$3.2 \cdot 10^{-3} K = 4$$

$$K = 1.25 \cdot 10^3$$

0 dB crossing  
phase margin

5

In Bode form

$$GH(s) = \frac{32 \cdot 1.25 \cdot 10^3}{10^{10}}$$

$$s \left(1 + \frac{s}{2}\right)^2 \left(1 + \frac{s}{100}\right) \left(1 + \frac{s}{800}\right)^2 \left(1 + \frac{s}{5000}\right)$$

$$\left(1 + \frac{s}{10}\right)^2 \left(1 + \frac{s}{40}\right) \left(1 + \frac{s}{500}\right)^2 \left(1 + \frac{s}{1000}\right) \left(1 + \frac{s}{2000}\right)$$





NOT STABLE  
UNSTABLE



# ROY CHALITH

5



